Modal Logic: Basics and Extensions
– Sheet 1: Propositional Logic –

Exercise 1 (3 Points, Declarative sentences)
Use \(\neg, \rightarrow, \land, \lor\) to express the following declarative sentences in propositional logic. (For each case describe the meaning of all predicates you use.)

1. If the sun shines today, then it won’t shine tomorrow.
2. If the barometer falls, then either it will rain or it will snow.
3. If you have read the lecture notes and if you have done the homework assignments, then you should be in a good shape for the first exam; otherwise, you will have a problem.

Exercise 2 (3 Points, Macros)
Recall Definition 1.11 (semantics of propositional logic). Write down the appropriate semantic clauses for the other connectives previously defined as macros (Definition 1.8).

Exercise 3 (2 Points, Tableaux method - rules)
For splitting the input formula into subformulae we need the following rules:

\[
\begin{align*}
\land &\vdash \frac{\phi \land \psi}{\phi} \\
\lor &\vdash \frac{\phi \lor \psi}{\phi} \\
\neg 1 &\vdash \frac{\psi}{\neg \neg \phi} \\
\neg 2 &\vdash \frac{\neg \neg \phi}{\psi}
\end{align*}
\]

Write down appropriate rules for \(\rightarrow\) and \(\leftrightarrow\).

Exercise 4 (3 Points, Possible valuations and models)
State all possible valuations of the formula

\[\varphi \equiv (\neg p \land q) \rightarrow (p \land (q \lor \neg r))\]

and indicate which of them are models of \(\varphi\).

Exercise 5 (4 Points, Models)
Consider a propositional language over exactly four constants, \(x_1, x_2, x_3, x_4\). How many models (i.e. valuations which satisfy the respective formula) are there for the following sentences?

1. \(\neg(\neg x_1 \lor x_2)\)
2. \(\neg(\neg x_1 \lor \neg x_2) \lor \neg(\neg x_2 \lor \neg x_3)\)
3. \(\neg(\neg(x_1 \land x_2) \rightarrow (x_1 \lor x_3 \lor x_4)) \lor \neg x_2\)
4. \(x_1 \leftrightarrow x_2 \leftrightarrow x_3\)
Exercise 6 (3 Points, Semantic concepts)

Decide whether each of the following sentences is valid, unsatisfiable, or neither of these and argue why!

1. \( x_1 \rightarrow x_2 \)
2. \( x_1 \lor x_2 \lor (x_1 \rightarrow x_2) \)
3. \( (x_2 \rightarrow x_1) \rightarrow (\neg x_2 \rightarrow \neg x_1) \)

Exercise 7 (8 Points, SPASS riddle)

You are a correspondent in Wongowongo. You have to report on the president election. Because you don’t know anything about the candidates you ask several people. Three of them are members of the either-or-party and three of them members of the consequence-party. These are the answers:

A: “surname Songo: city Rongo” and “consequence-party: first name Mongo”
B: “either-or-party: older” and “clan Bongo: surname Gongo”
C: “first name Dongo: at the poll behind” and “first name Dongo: younger”
D: “clan Bongo: at the poll ahead” and “city Longo: younger”
E: “first name Mongo: at the poll behind” and “clan Nongo: younger”
F: “consequence-party: clan Nongo” and “consequence-party: surname Gongo”

The members of the either-or-party (A, B, C) always give one correct answer and one false statement. In contrast, the fans of the consequence-party give only correct answers or their answers are completely wrong.

Who are the candidates and what kind of information can you gain?

Write a program for spass (http://www.spass-prover.org/) and submit the paper version.