Exercise 1 (4 Points, Optimality of Uniform-cost - and Breadth-first search)
Prove that uniform-cost search (with arbitrary step costs) and breadth-first search with constant step costs are optimal when used with the Graph-Search algorithm.

Exercise 2 (6 Points, Comparison of search algorithms)
Consider a finite tree of depth \( d + 1 \) and branching factor \( b \). Suppose the shallowest goal node is at depth \( g \leq d \) (\( g \) is given and fixed). What is the minimal and maximal number of nodes that might be generated by

1. depth-first search with depth bound equal to \( d \).
2. breadth-first search?
3. depth-first iterative deepening search?

Exercise 3 (6 Points, Best-first search)
We consider Best-first search with Tree-search as underlying search engine. We assume as usual a finite branching factor and that step costs are higher than a fixed constant.

(a) Prove or disprove the following two statements with respect to : (1) Best-first search is optimal; (2) best-first search is complete.

(b) We now consider Best-first search together with an admissible heuristics; i.e. \( f(n) := h(n) \) where \( h \) is admissible. Reconsider the statements in (a).

Exercise 4 (4 Points, A* search)
Trace the operation of A* search applied to the problem of getting to Bucharest from Lugoj using the straight-line distance heuristic (the state space and the heuristic are given in Example 2.6). That is, show in which order nodes are expanded, and state the costs of the actual optimal path.

Exercise 5 (3 Points, Admissibility and Graph-search)
Show that admissibility of \( h \) is insufficient to ensure that A* is optimal together with Graph-search.

Exercise 6 (6 Points, consistency and admissibility)
Let \( h \) be a heuristic function and \( f(n) = g(n) + h(n) \) the evaluation function of A* search. Prove or disprove the following statements (give proofs or counterexamples):

1. Admissibility of \( h \) implies consistency of \( h \).
2. Consistency of \( h \) implies admissibility of \( h \).