Exercise 1 (4 Points, 0L-System)
Construct an 0L-System $G$ which generates the language $L := \{a^n \mid n \geq 0, n \neq 3\}$ and prove that $L = L(G)$.

Exercise 2 (4 Points, Lindenmayer)
Let $\text{Fib} \subseteq \mathbb{N}_0$ be the set of Fibonacci numbers and $\text{Fib}^0 := \text{Fib} \setminus \{0\}$.
Construct a DOL grammar $G = (\Sigma, w, R)$ such that $G(L)$ and $\text{Fib}^0$ are isomorphic.
That is, show that there is a bijective function $f : G(L) \rightarrow \text{Fib}^0$.

Exercise 3 (3 Points, Word problem for LBA’s)
Let an LBA $M$ be given. Assume $M$ has $k$ states and $m$ tape symbols. How many different configurations exist at most for an input word $w$ with $|w| = n$?
Show that the halting problem for LBA is decidable.

Exercise 4 (4 Points, LBA vs DTM)
Show that it is undecidable (by constructing a suitable reduction of the halting problem) to determine whether a TM $M$ (as input) is in fact an LBA.
Discuss the difference of this problem to the following: Decide whether the accepted language of a given TM $M$ is context-sensitive.

Exercise 5 (6 Points, PCP$_1$, \{0, 1\}-PCP)
We consider variants of PCP.
1. Prove Lemma 2.9: The PCP$_1$ (PCP over a one alphabet $|\Sigma| = 1$) is decidable.
2. Prove Lemma 2.11: PCP can be reduced to \{0, 1\}-PCP.
3. Show that the PCP: $\{\langle a^2b, a^2 \rangle_1, \langle a, ba^2 \rangle_2\}$ has no solution.

Exercise 6 (6 Points, Reduction)
For a given TM $M$, show by reduction that the following problems are undecidable:
1. $L(M)$ is context-sensitive.
2. $L(M)$ is context-free.
3. $L(M)$ is regular.
4. $L(M)$ has exactly 17 elements.