Logic and Verification
– Sheet 2: Sentential Logic and Verification 1 –

Exercise 1 (8 Points, Resolution Calculus)
Consider the theory $T = \{ a \lor b, (c \rightarrow (d \rightarrow a)), \neg a \leftrightarrow \neg b\}$.

(a) Use the Resolution calculus from Definition 2.36 to show that this theory entails $b \lor d$. Explain all the steps.

(b) Show that $T$ does not entail $c$.

(c) Is the following true: $T \vdash_{\text{Res}} b \lor d$? Prove or disprove.

(d) Is the following true: $T \vdash_{\text{Res}} a$? Prove or disprove.

Exercise 2 (6 Points, Verification)
Consider the following sequential hardware circuits:

```
NOT   AND   OR
r1    AND   OR
x1    y1    y1
x2    y2    r2
```

(a) Give the transition systems for both hardware circuits. Assume that the initial values of the registers are $r_1 = 0$ and $r_2 = 1$.

(b) Determine the reachable part of the transition system of the synchronous product of the transition systems from (a).

Exercise 3 (12 Points, Verification)
The following program is a mutual exclusion protocol for two processes due to Pnueli. There is a single shared variable $s$ which is either 0 or 1, and initially 1. Besides, each process has a local Boolean variable $y$ that initially equals 0. The program text for process $P_i (i = 0, 1)$ is as follows:

```
while true do
  Noncritical section
  $(y, s) := (1, i)$;
  wait until $((y_1-i = 0) \lor s \neq i)$;
  Critical section
  $y_i := 0$
end while
```
Here, the statement \((y_i, s) := (1, i)\); is a multiple assignment in which variable \(y_i := 1\) and \(s := i\) is a single, atomic step.

a) Define the program graph of a process in Pnueli’s algorithm.

b) Determine the transition system for each process.

c) Construct their parallel composition. (Hint: Construct interleaving of program graphs and then the unfolding into a transition system. Compare with Figures 3.21 and 3.22 on Slides 201 and 202.)

d) Check whether the algorithm ensures mutual exclusion.

e) Check whether the algorithm ensures starvation freedom.

**Hint**: The last two questions may be answered by inspecting the transition system.

**Exercise 4 (0 Points, Extra (15P): Compactness)**

Given a (possibly countably infinite) undirected graph. Show that it can be colored (each node gets a color and connected nodes get different colors) with at most four colors if and only if each finite subgraph can be colored with at most four colors.

**Hint**: Use the compactness theorem (Corollary 2.31) and choose the language and the theory \(T\) appropriately.