Exercise 1 (10 Points, LTL Semantics)

Consider the following transition system over the set of atomic propositions \{a, b\}:

1. In which states are the following LTL formulae true:
   (a) $\Box a$
   (b) $\Box \Box a$
   (c) $\Diamond b$
   (d) $\Diamond F b$
   (e) $b \mathcal{U} a$
   (f) $\neg \Diamond F \neg b$

2. Which of those formulae (from 1.) hold in the transition system?

Exercise 2 (10 Points, LTL properties)

Suppose we have two users, Peter and Betsy, and a single printer device Printer. Both
users perform several tasks, and every now and then they want to print their results
on the Printer. Since there is only a single printer, only one user can print a job at a
time. Suppose we have the following atomic propositions for Peter at our disposal:

- \texttt{Peter.request ::= } indicates that Peter requests usage of the printer;
- \texttt{Peter.use ::=} indicates that Peter uses the printer;
- \texttt{Peter.release ::= } indicates that Peter releases the printer.

For Betsy, similar predicates are defined. Specify in LTL the following properties:

(a) Mutual exclusion, i.e., only one user at a time can use the printer.
(b) Finite time of usage, i.e., a user can print only for a finite amount of time.
(c) Absence of individual starvation, i.e., if a user wants to print something, he/she
   is eventually able to do so.
(d) Absence of blocking, i.e., a user cannot print an infinite number of times without
   releasing the printer in between.
(e) Alternating access, i.e., (i) users cannot print simultaneously and (ii) after having
   printed, each user must wait for the other to print before she can print again.
Exercise 3 (8 Points, LTL formulae)

Are the following LTL formulae valid, not valid but satisfiable or unsatisfiable?

1. \( X(a \lor Fa) \rightarrow Fa \),
2. \( (Ga)U (Fb) \rightarrow G(aU (Fb)) \),
3. \( GG(\varphi \lor \neg \psi) \leftrightarrow \neg (F(\neg \varphi \land \psi)) \),
4. \( ((\varphi U \psi)U \psi) \leftrightarrow (\varphi U \psi) \).

Exercise 4 (8 Points, LTL and FO \((\leq)\))

Express the following LTL formulae as FOL formulae in FO \((\leq)\) and bring them in prenex normal form.

1. \( (XX\neg p) \rightarrow GF(\neg p) \),
2. \( (XX\neg p)U (Gp) \)

Exercise 5 (0 Points, Extra (15P): Compactness)

Given a (possibly countably infinite) undirected graph. Show that it can be colored (each node gets a color and connected nodes get different colors) with at most four colors if and only if each finite subgraph can be colored with at most four colors.

Hint: Use the compactness theorem (Corollary 2.31) and choose the language and the theory \(T\) appropriately.