Exercise 1 (4 Points, Tiling)

Show that finite plane tiling is recursively enumerable while plane tiling with restricted origin is co-recursively enumerable.

Exercise 2 (5 Points, Makanin)

Does it make a difference to consider a system of word equations or just a single equation? What about the same question for quadratic equations (system of equations)? Apply the algorithm for solving quadratic word equations to the equation $xauzau = yzbxaaby$.

Exercise 3 (4 Points, Ackermann)

Prove Theorem 2.39: for all $f \in \mathcal{E}^n$ there is $c_f$ such that $\forall T$:

$$f(T) \leq \text{Ack}_{n+1}(\max\{2, T\}, c_f)$$

Exercise 4 (6 Points, Ackermann)

1. Let $n_0 \in \mathbb{N}$. Is $\text{Ack}_{n+1}(x, n_0) : \mathbb{N} \rightarrow \mathbb{N}$ contained in $\mathcal{E}_n$? Prove or disprove.

2. Discuss why the Ackermann-Péter function $AP$ is welldefined. Give a (good) lower bound on $AP(4, y)$.

Exercise 5 (8 Points, Grzegorczyk-Hierarchy)

1. Consider the function $fib : \mathbb{N} \rightarrow \mathbb{N}$, which assigns to a number $i$ the $i+1$th Fibonacci number. Show that (1) $fib \in \mathcal{E}^3$, (2) $fib(2n) \geq 2^n$ for $n \geq 0$ and (3) $fib \notin \mathcal{E}^2$.

2. Show that $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(0) = 1$, $f(n + 1) = 2^{f(n)}$ is not in $\mathcal{E}$.

3. Show that $f : \mathbb{N}^2 \rightarrow \mathbb{N}; (x, y) \mapsto x^{y+1}$ is in $\mathcal{E}$. 