Logic and Verification
– Sheet 3: Verification + FOL –

Exercise 1 (10 Points, LTL Semantics)
Consider the following transition system over the set of atomic propositions \{a, b\}:

\[
\begin{array}{c}
&s_1 & \{a, b\} \\
\downarrow & \downarrow & \downarrow \\
&s_2 & \{a\} \\
\uparrow & \uparrow & \uparrow \\
&s_3 & \{a, b\} \\
\downarrow & \downarrow & \downarrow \\
&s_4 & \{b\}
\end{array}
\]

1. In which states are the following LTL formulae true:
   (a) Xa
   (b) XXa
   (c) Gb
   (d) GFb
   (e) bUa
   (f) ¬FG¬b

2. Which of those formulae (from 1.) hold in the transition system?

Exercise 2 (6 Points, LTL properties)
Suppose we have two users, Peter and Betsy, and a single printer device Printer. Both users perform several tasks, and every now and then they want to print their results on the Printer. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for Peter at our disposal:

- Peter.request ::= indicates that Peter requests usage of the printer;
- Peter.use ::= indicates that Peter uses the printer;
- Peter.release ::= indicates that Peter releases the printer.

For Betsy, similar predicates are defined. Specify in LTL the following properties:

(a) Mutual exclusion, i.e., only one user at a time can use the printer.

(b) Absence of individual starvation, i.e., if a user wants to print something, he/she is eventually able to do so.

(c) Alternating access, i.e., (i) users cannot print simultaneously and (ii) after having printed, each user must wait for the other to print before she can print again.
Exercise 3 (6 Points, LTL operators)
Let $\varphi$ and $\psi$ be LTL formulae. Consider the following new operators:

1. “At next” $\varphi N \psi$: at the next time where $\psi$ holds, $\varphi$ also holds.
2. “While” $\varphi W \psi$: $\varphi$ holds at least as long as $\psi$ does.
3. “Before” $\varphi B \psi$: if $\psi$ holds sometime, $\varphi$ does so before.

Make the definitions of these informally explained operators precise by providing LTL formulae that formalize their intuitive meanings.

Exercise 4 (6 Points, LTL formulae)
Are the following LTL formulae, valid, not valid but satisfiable or unsatisfiable?

1. $(G \neg (a \land b)) \land (a U b) \land (b \rightarrow a)$
2. $X(a \lor Fa) \rightarrow FFa$
3. $GG(\psi \rightarrow \varphi) \leftrightarrow \neg (F(\neg \varphi \land \psi))$

Exercise 5 (8 Points, LTL and FO ($\leq$))
Express the following LTL formulae as FOL formulae in FO ($\leq$) and bring them in prenex normal form.

1. $(XXp) \rightarrow GF(\neg p)$
2. $(XXXp)U(G \neg p)$

Exercise 6 (0 Points, Extra (15P): LTL expressibility)
We have seen that many interesting properties can be expressed with LTL formulae. In this exercise, we ask for a proof that a certain property cannot be expressed with any LTL formula. We consider Example 3.39 of the lecture (Slide 272). There is no LTL formula to express that “relation $r$ is true at all even states (evenness)”. Odd states do not matter: $r$ can be true or not. Prove this statement.

Hint: Find two infinite series of models (paths) $\lambda_j$, $\lambda_j'$, $j = 0, 1, \ldots$ such that the following holds: (1) for all $j$ the property evenness is either true at $\lambda_j$ or at $\lambda_j'$, and (2) for all $j$ and for all LTL formulae $\varphi$ of depth at most $j$, $\lambda_j$ and $\lambda_j'$ cannot be distinguished (i.e. $\lambda_j \models \varphi$ if and only if $\lambda_j' \models \varphi$ for all such $\varphi$). Define depth of a LTL formula so that it all works.

Comment: In fact, LTL can express all properties of *-free $\omega$-regular languages. The property above is not of this kind: one needs the Kleene-* to express it for infinite words.