TU Clausthal

Department of Informatics

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Artificial Intelligence

– Sheet 5: Neural Nets and Sentential Logic –

Date: 17. May 2011

Points:

____________ of 26

Group / Tutor:

Name(s) & Matr. no.:

Exercise 1 (8 Points, Perceptron)

Indicate which of the following Boolean functions $\{0,1\}^3 \to \{0,1\}$ (i.e., in particular $1+1=0, \bar{1}=0$ and $\bar{0}=1$) of three input variables can be realized by a single perceptron. If it is possible to represent a function by a perceptron draw it and calculate the corresponding weights. In the case that a perceptron does not exist show why not.

1. $(x_1, x_2, x_3) \mapsto x_1 x_2 x_3$
2. $(x_1, x_2, x_3) \mapsto x_1 + x_2 + x_3$
3. $(x_1, x_2, x_3) \mapsto (x_1 x_2 x_3) + (\bar{x}_1 \bar{x}_2 \bar{x}_3)$
4. $(x_1, x_2, x_3) \mapsto 1$

Exercise 2 (3 Points, Linear activation functions)

Consider a (feed-forward) neural network with linear activation functions and n hidden layers, i.e. each neuron i has an activation function $in_i \mapsto c_i in_i + d_i$. Show that this network is equivalent to a network with no hidden layers. (Use the notations introduced in the lecture!)

In equation (a) the output of N can be considered as function of the activation values a_i, (that is, activation values of the units in the n^{th} hidden layer). Extending the formula, we can get equation (b), where the terms $\sum_{i=0}^{k} W_i W_{j,i} c_i$ and $c_o(\sum_{i=0}^{k} W_i d_i) + d_o$ depend only on weights and constants, and they can be considered constants as well (they do not depend on activation values). So we call the result of these terms $W'_j a_j$ and d'_o respectively. Now we arrive to equation (c), which look similar to equation (a) but is defined in terms of the activation values a_j that is, activation values of the units in the $n-1^{th}$ hidden layer, and has no explicit reference to the values a_i of the n^{th} hidden layer. This procedure can be iterated until a network with no hidden layers is obtained.

Exercise 3 (4 Points, Convergence Theorem)

In the proof of Theorem 4.6 (Rosenblatt's Convergence Theorem) it is claimed that

$$\hat{w} \cdot \overrightarrow{I} > 0 \text{ for all } \overrightarrow{I} \in \mathbf{I}_{\text{pos}} \cup -\mathbf{I}_{\text{neg}}$$

if $\hat{w}$ represents a solution. Prove this claim!

Exercise 4 (2 Points, Tautologies)

State two tautologies that contain the constants $Breathe$ and Die. How many tautologies can be built with these variables and the logical connectives? How many of these tautologies are equivalent to each other?

Exercise 5 (4 Points, Models)

Consider a vocabulary with only four propositions, A, B, C and D. How many models are there for the following sentences?

To be submitted:

08. June 2011
before class

1. $(A \wedge B) \vee (B \wedge C)$
2. $A \leftrightarrow B \leftrightarrow C$
3. $((A \wedge B) \rightarrow (A \vee C \vee D)) \wedge B$
4. $\neg(A \rightarrow A)$

Exercise 6 (3 Points, Validity, truth, and satisfiability)

Decide whether each of the following sentences is valid, unsatisfiable, or neither of these.

(a) $Smoke \rightarrow Fire$

(b) $(Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire)$

(c) $Big \vee Dumb \vee (Big \rightarrow Dumb)$

Exercise 7 (2 Points, CNF)

Determine the smallest set of clauses equivalent to the following formula:

$$\neg\big(((A \vee \neg B) \rightarrow C) \rightarrow (A \wedge C)\big)$$