Exercise 1 (6 Points, Space Constructible)

1. Each space constructible function $S(n)$ with $S(n) \geq n$ is also fully space constructible.

2. If a language $L$ is accepted by a $S(n)$ space bounded TM with $S(n) \geq \log_2(n)$, then $L$ is accepted by a $S(n)$ space bounded TM that halts on all inputs.

Exercise 2 (8 Points, Fully Space Constructible)
Show that $\text{DTIME}(f(n) \log n) \subseteq \text{DSPACE}(f(n))$, if $f(n)$ is fully space constructible.

Exercise 3 (10 Points, Blum’s speed-up)
The speed-up theorem tells us that for large enough recursive functions $r$ there is a language $L$ s.t. there is a sequence $M_1, M_2, \ldots, M_i$ of TM’s accepting $L$ with: the space used by $M_i$ is at least $r$ applied to the space used by $M_{i+1}$.
Prove that speed-up is not effective: If for each TM accepting $L$ there is a $M_i$ on the list using less space, then the list is not r.e.

Exercise 4 (10 Points, Recursive Languages are not contained in a complexity class)
Given any total recursive time-bound $T(n)$ there is a recursive language $L$ that is not in $\text{DTIME}(T(n))$. 

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