



Artificial Intelligence

– Sheet 5: Neural Nets and Sentential Logic –

Date: 23. May 2012

Exercise 1 (8 Points, Perceptron)

Indicate which of the following Boolean functions $\{0, 1\}^3 \rightarrow \{0, 1\}$ (i.e., in particular $1 + 1 = 0$, $\bar{1} = 0$ and $\bar{0} = 1$) of three input variables can be realized by a single perceptron. If it is possible to represent a function by a perceptron draw it and calculate the corresponding weights. In the case that a perceptron does not exist show why not.

1. $(x_1, x_2, x_3) \mapsto x_1x_2x_3$
2. $(x_1, x_2, x_3) \mapsto x_1 + x_2 + x_3$
3. $(x_1, x_2, x_3) \mapsto (x_1x_2x_3) + (\bar{x}_1\bar{x}_2\bar{x}_3)$
4. $(x_1, x_2, x_3) \mapsto 1$

Points:

_____ of 26

Group/Tutor:

Name(s) & Matr. no.:

Exercise 2 (3 Points, Linear activation functions)

Consider a (feed-forward) neural network with linear activation functions and n hidden layers, i.e. each neuron i has an activation function $in_i \mapsto c_i in_i + d_i$. Show that this network is equivalent to a network with no hidden layers. (Use the notations introduced in the lecture!)

Exercise 3 (4 Points, Convergence Theorem)

In the proof of Theorem 4.6 (Rosenblatt's Convergence Theorem) it is claimed that

$$\hat{w} \cdot \vec{I} > 0 \text{ for all } \vec{I} \in \mathbf{I}_{\text{pos}} \cup -\mathbf{I}_{\text{neg}}$$

if \hat{w} represents a solution. Prove this claim!

Exercise 4 (2 Points, Tautologies)

State two tautologies that contain the constants *Breathe* and *Die*. How many tautologies can be built with these variables and the logical connectives? How many of these tautologies are equivalent to each other?

Exercise 5 (4 Points, Models)

Consider a vocabulary with only four propositions, A , B , C and D . How many models are there for the following sentences?

1. $(A \wedge B) \vee (B \wedge C)$
2. $A \leftrightarrow B \leftrightarrow C$
3. $((A \wedge B) \rightarrow (A \vee C \vee D)) \wedge B$
4. $\neg(A \rightarrow A)$

To be submitted:

20. June 2012
before class



Exercise 6 (3 Points, Validity, truth, and satisfiability)

Decide whether each of the following sentences is valid, unsatisfiable, or neither of these.

- (a) $Smoke \rightarrow Fire$
- (b) $(Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire)$
- (c) $Big \vee Dumb \vee (Big \rightarrow Dumb)$

Exercise 7 (2 Points, CNF)

Determine the smallest set of clauses equivalent to the following formula:

$$\neg(((A \vee \neg B) \rightarrow C) \rightarrow (A \wedge C))$$