Exercise 1 (10 Points, Perceptron)

Which of the following Boolean functions of three input variables (from \{0, 1\}^3 into \{0, 1\}, in particular 1 + 1 = 0, 1 = 0 and 0 = 1) can be realized by a single perceptron? If it is possible to represent the function, determine the perceptron and calculate the corresponding weights. In the case the function is not representable, show why this is not the case.

1. \((x_1, x_2, x_3) \mapsto \overline{x}_1 \overline{x}_2 \overline{x}_3\),
2. \((x_1, x_2, x_3) \mapsto (x_1 + x_2)x_3\),
3. \((x_1, x_2, x_3) \mapsto x_1 x_3 + x_2 x_3\),
4. \((x_1, x_2, x_3) \mapsto x_1 x_2 + x_2 x_3 + x_3 x_1\).

Exercise 2 (3 Points, Tautologies)

State two tautologies that contain the propositions Study and Enjoy. How many different tautologies can be built with these variables and the logical connectives? How many of these tautologies are equivalent to each other?

Exercise 3 (4 Points, Validity, truth, and satisfiability)

Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(a) \((New \rightarrow Good) \rightarrow New\)
(b) \((New \rightarrow Good) \rightarrow (\neg Good \rightarrow \neg New)\)
(c) \((Red \land \neg Round) \lor (Round \rightarrow Red)\)

Exercise 4 (12 Points, Entailment)

In the following, you are given two knowledge bases involving unicorns. Consider for each of the knowledge bases the questions: (1) Is the unicorn mythical? (2) Is the unicorn magical? (3) Is the unicorn horned? Prove which of these properties hold and prove which of these properties do not hold (i.e. are not entailed).

(a) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

(b) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.
Exercise 5 (4 Points, Models)
Consider a vocabulary with exactly four variables, $x_1$, $x_2$, $x_3$ and $x_4$. How many models are there for the following sentences?

1. $\neg(\neg x_1 \lor x_1)$
2. $\neg(\neg x_1 \lor \neg x_2) \lor \neg(\neg x_2 \lor \neg x_3)$
3. $x_1 \leftrightarrow x_2 \leftrightarrow x_3$
4. $\neg((x_1 \land x_2) \rightarrow (x_1 \lor x_3 \lor x_4)) \lor \neg x_2$