Exercise 1 (4 Points, Weakest precondition)
Calculate the weakest precondition of the following program with respect to the postcondition \( \{ 4 \leq y < 22 \} \).

\[
x := 3y;
\text{if} \ (x < 10) \ \{ 
y := 3x - 9 
\} \ \text{else} \ \{ 
y := 2x + 1
\};
y := y - 5
\]

Exercise 2 (10 Points, Partial correctness)
Consider the following programs and use the annotation calculus to prove that the program is partially correct regarding the given pre and postconditions (do not forget the proof obligations).

(a) \if (x > y) \{ 
\quad z := x;
\} \ \text{else} \ \{ 
\quad z := y
\}
Precondition \( \top \), postcondition \( z = \max\{x, y\} \).

(b) \z := 0;
\text{while} \ x > 0 \ { 
\quad z := z + x;
\quad x := x - 1
}\)
Precondition \( x = x_0 \land x \geq 0 \), postcondition \( z = \frac{x_0(x_0 + 1)}{2} \).

Exercise 3 (10 Points, Total correctness)
Consider the following program \( \text{Pow}(x) \):

\[
y := 1;
a := 0;
\text{while} \ (a < x) \ \{ 
y := 2y;
a := a + 1
\}
\]

(a) Find a suitable variant for the loop and explain it.

(b) Use the annotation calculus to prove that \( \vdash^t \{ x \geq 0 \} \text{Pow}\{ y = 2^x \} \)
Exercise 4 (10 Points, Invariant)

(a) Determine a suitable invariant for the program and show, using the annotation calculus, that it is partial correct under the precondition \( z \geq 0 \) and the postcondition \( x = y \times z \). Explain your solution.

\[
\begin{align*}
x & := 0; \\
a & := 0; \\
\text{while } (a < z) \\
& \quad \{ \\
& \quad \quad x := x + y; \\
& \quad \quad a := a + 1; \\
& \quad \}
\end{align*}
\]

(b) Is the program in (a) also totally correct (with the same pre and postcondition)? Prove your answer.