Exercise 1 (5 Points, Weakest precondition)
Calculate the weakest precondition of the following program with respect to the postcondition \( \{ 5 \leq y < 23 \} \).

\[
x := 2y; \\
\text{if } (x > 13) \{ \\
\quad y := 4x - 13; \\
\} \text{ else } \{ \\
\quad y := 2x - 8; \\
\}; \\
y := y + 1
\]

Exercise 2 (10 Points, Partial correctness)
Consider the following programs and use the annotation calculus to prove that each one is partially correct regarding the given pre and postconditions (make sure your annotation is valid).

(a) \( \text{if } (x > y) \{ \\
\quad z := x; \\
\} \text{ else } \{ \\
\quad z := y; \\
\}
\) Precondition \( \top \), postcondition \( z = \max\{x, y\} \).

(b) \( y := 3; \\
z := 1; \\
\text{while } (y > 0) \{ \\
\quad z := 2x; \\
\quad y := y - 1; \\
\}
\) Precondition \( x \geq 0 \), postcondition \( z = x^3 \).
Exercise 3 (12 Points, Total correctness)

Consider the following program Div:

```plaintext
z := 0;
a := x;
while (a ≥ y) {
    a := a − y;
    z := z + 1
}
```

Use the annotation calculus to prove that \( \vdash t \{ x ≥ 0 ∧ y > 0 \} \text{Div} \{ z = x/y \} \)
(where “/” denotes simple integer division).

Exercise 4 (10 Points, Soundness)

Prove Theorem 5.48: “The Hoare calculus is sound.”

Exercise 5 (0 Points, Extra (15P): A tricky algorithm)

Consider the following program P (it uses x as input and produces the output y):

```plaintext
i := 0;
y := 0;
j := 0;
k := 0;
while (i ≠ x) {
    i := i + 1;
j := 2i;
k := y + j;
y := k − 1;
}
```

a) Which function \( f : \mathbb{N} → \mathbb{N} \) does P compute?

b) Formulate appropriate pre and postconditions and prove total correctness using the annotation calculus.