TU Clausthal

Department of Informatics

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Artificial Intelligence

– Sheet 6: Sentential Logic and Hoare Calculus –

Date: 01. June 2011

Points:

______________ of 28

Group / Tutor:

Name(s) & Matr. no.:

Exercise 1 (5 Points, Boolean connectives)

In the lecture, we have introduced the Boolean connectives $C := \{\neg, \vee, \wedge, \rightarrow\}$.

(a) (2 Points) Show that the Boolean connectives $\{\neg, \wedge\}$ are sufficient. That is, show that each connective from C can be defined from just these two.

(b) (3 Points) Consider the binary Boolean function $| : \{0,1\}^2 \rightarrow \{0,1\}$ defined by

$$(1,1) \mapsto 0, \quad (0,1) \mapsto 1, \quad (1,0) \mapsto 1, \quad (0,0) \mapsto 1$$

Is it possible to define all Boolean connectives from C just by "|" ? Prove or disprove!

Exercise 2 (7 Points, Resolution)

(a) (4 points) Given the description below, can you prove - *using resolution* - that the unicorn is mythical? How about magical? Horned? Explain your proceeding!

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal *or* a mammal, then it is horned. The unicorn is magical if it is horned.

(b) (3 points) Consider now a slightly modified description:

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is *either* immortal *or* a mammal, then it is horned. The unicorn is magical if it is horned.

Prove that nothing can be said about the unicorn.

Exercise 3 (5 Points, Hilbert Calculus)

(a) (3 points) Use the Hilbert calculus from Definition 4.12 to prove the formula (schema) $\varphi \rightarrow \varphi$.

(b) (2 points) Consider the theory $T = \{(c \rightarrow d), ((d \rightarrow a) \rightarrow (b \rightarrow c)), (c \vee d), b, a\}$.

1. Prove that $T \vdash c$ by means of the Hilbert calculus from Definition 4.12.
2. Prove that $T \models c$. (It is not allowed to use the completeness result of the calculus!)

(Explain all steps!)

To be submitted:

22. June 2011
before class

Exercise 4 (5 Points, Partial correctness)

Consider the following programs and use the annotation calculus to prove that the program is *partially* correct regarding the given pre and postconditions.

(a)
```
if (x > y) {
    z := x;
} else {
    z := y
}
```
Precondition $\top$, postcondition $z = \max\{x, y\}$.

(b)
```
z := 0;
while x > 0 {
    z := z + x;
    x := x − 1
}
```
Precondition $x = x_0 \wedge x \geq 0$, postcondition $z = \frac{x_0(x_0+1)}{2}$.

(Note, that also the proof obligations have to be verified!)

Exercise 5 (3 Points, Total correctness)

Consider the following program Mult:

```
a := 0;
z := 0;
while (a ≠ y) {
        z := z + x;
        a := a + 1
    }
```

Use the annotation calculus to prove that $\vdash^t \{y \geq 0\}$Mult$\{z = xy\}$

Exercise 6 (3 Points, Weakest precondition)

Calculate the weakest precondition of the following program with respect to the postcondition $\{4 \leq y < 22\}$.

```
x := 3y;
if (x < 10) {
        y := 3x − 9
} else {
        y := 2x + 1
};
y := y − 5
```