TU Clausthal

Department of Informatics

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Artificial Intelligence

– Sheet 6: Sentential Logic and Hoare Calculus –

Date: 19. June 2012

Points:

__________ of 28

Group / Tutor:

Name(s) & Matr. no.:

To be submitted:

03. July 2012
before class

Exercise 1 (5 Points, Boolean connectives)

In the lecture, we have introduced the Boolean connectives $C := \{\neg, \vee, \wedge, \rightarrow\}$.

(a) (2 Points) Show that the Boolean connectives $\{\neg, \vee\}$ are sufficient. That is, show that each connective from C can be defined from just these two.

(b) (3 Points) Consider the binary Boolean function $| : \{0,1\}^2 \rightarrow \{0,1\}$ defined by

$$(1,1) \mapsto 0, \quad (0,1) \mapsto 0, \quad (1,0) \mapsto 0, \quad (0,0) \mapsto 1$$

Is it possible to define all Boolean connectives from C just by "|" ? Prove or disprove!

Exercise 2 (7 Points, Resolution)

(a) (4 points) Given the description below, can you prove - *using resolution* - that the player is invincible? How about good? fast? Explain your proceeding!

> If the player is invincible, then he is the winner of the match, but if he is not invincible, then he is a strong loser. If the player is the winner *or* he is strong, then he is fast. The player is good if he is fast.

(b) (3 points) Consider now a slightly modified description:

> If the player is invincible, then he is the winner of the match, but if he is not invincible, then he is a strong loser. If the player is *either* the winner *or* he is strong, then he is fast. The player is good if he is fast.

Prove that nothing can be said about the player.

Exercise 3 (5 Points, Hilbert Calculus)

(a) (3 points) Use the Hilbert calculus from Definition 5.12 to prove the formula (schema) $\varphi \rightarrow \varphi$.

(b) (2 points) Consider the theory $T = \{((a \rightarrow b) \rightarrow (c \rightarrow d)), (\neg e \vee a), (d \rightarrow a), b, c\}$.

1. Prove that $T \vdash d$ by means of the Hilbert calculus from Definition 4.12.
2. Prove that $T \models d$. (It is not allowed to use the completeness result of the calculus!)

(Explain all steps!)

Exercise 4 (5 Points, Partial correctness)

Consider the following programs and use the annotation calculus to prove that the program is *partially* correct regarding the given pre and postconditions.

(a)
```
if (z ≥ y) {
    x := z;
} else {
    x := y;
}
```
Precondition $\top$, postcondition $x = \max\{y, z\}$.

(b)
```
z := x;
while z > 0 {
    z := z − 1;
    x := x + z;
}
```
Precondition $x = x_0 \wedge x \geq 0$, postcondition $x = \frac{x_0(x_0+1)}{2}$.

(Note, that also the proof obligations have to be verified!)

Exercise 5 (3 Points, Total correctness)

Consider the following program Pow:

```
a := 0;
z := 1;
while (a ≠ x) {
        z := z ∗ y;
        a := a + 1;
    }
```

Use the annotation calculus to prove that $\vdash^t \{x \geq 0\}$Pow$\{z = y^x\}$

Exercise 6 (3 Points, Weakest precondition)

Calculate the weakest precondition of the following program with respect to the postcondition $\{3 \leq y < 42\}$.

```
x := 2y;
if (x > 4) {
        y := 2x − 4
} else {
        y := 4x + 1
};
y := 2y + 1
```