TU Clausthal

Department of Informatics

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Artificial Intelligence

– Sheet 7: FOL and Provers –

Date: 24. June 2011

Points:

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Group / Tutor:

Name(s) & Matr. no.:

Exercise 1 (5 Points, Logical formalization)

Translate the following sentences into predicate logic. *Specify* the used predicate and function symbols and give their intended semantic meaning (by stating the universe and the interpretation function).

1. Every football team has a goal keeper.
2. If the Germans beat the Portuguese, then the Germans do not lose to every football team.
3. The Portuguese beat some team, which beat the Germans.
4. All blue symbols are in the box.
5. Only blue symbols are in the right box.

Exercise 2 (6 Points, Interpretations)

Let the formula

$$\varphi \equiv \forall x \forall y \forall z (p(x,y) \wedge p(y,z) \rightarrow p(x,z))$$

be given.
Which of the following structures $A = (U, I)$ are models for φ? *Prove* or *disprove*!

1. $U = \mathbb{N}$, $I(p) = \{(m,n) \mid m,n \in \mathbb{N}, m < n\}$,
2. $U = \mathbb{N}$, $I(p) = \{(m,m+1) \mid m \in \mathbb{N}\}$,
3. $U = \mathcal{P}(\mathbb{N})$, $I(p) = \{(A,B) \mid A,B \subseteq \mathbb{N}, A \subseteq B\}$.

Exercise 3 (6 Points, Limitations of FOL)

There is no formula that is true exactly in those interpretations with infinitely many individuals. (You may assume the equality symbol "$\doteq$" is available). Give a formal proof for this statement! (Hint: Compactness Theorem.)

Exercise 4 (3 Points, Expressiveness)

Let $\exists! \varphi(x)$ mean that there exists a *unique* x such that $\varphi(x)$. Rewrite this formula using only the usual operators and quantifiers from first-order logic with equality (i.e, $\exists$, $\forall$, $\doteq$, $\neg$, $\wedge$, and $\vee$).

To be submitted:

06. July 2011
before class

Exercise 5 (4 Points, Resolution)

Use resolution to prove that the following clause set is unsatisfiable:

$$\begin{aligned}&\{\{P(a,x,f(y)),P(a,z,f(h(b))),\neg Q(y,z)\},\\&\{\neg Q(h(b),w),H(w,a)\},\\&\{\neg P(a,w,f(h(b))),H(x,a)\},\\&\{P(a,u,f(h(u))),H(u,a),Q(h(b),b)\},\\&\{\neg H(v,a)\}\}\end{aligned}$$

Exercise 6 (4 Points, Resolution)

Formalize the following statements in FOL:

1. The Professor is happy if all his students like logic.
2. The Professor is happy if he has no students.

Use resolution to show that the statement 2 follows from 1.