Remarks:

- For this exercise, you will need SWI-Prolog¹.
- For exercise 1-3, you need to send your solutions as source files to
  - Göttingen: maria.moloci@gwdg.de
  - CLZ: ta10@tu-clausthal.de.
- Make sure your source files do properly compile and run!
- Document and explain your code!

Exercise 1 (8 Points, Binomial coefficient)

(a) Write a predicate \texttt{binom/3} that takes two integers \(k, n \in \mathbb{Z}, k \geq 0\) and calculates the binomial coefficient \(\binom{n}{k}\). For example:

\begin{verbatim}
?- binom(3,2,X).
X = 3.
true.
?- binom(5,2,X).
X = 10.
true.
\end{verbatim}

(b) Now assume additionally that \(n \geq 0\) and \(n \geq k\). Use the predicate \texttt{fac/2} from the lecture (Ex. 7.74) to calculate the binomial coefficient. Let your predicate \texttt{binom/3} dynamically switch between both procedures, s.t. the one including \texttt{fac/2} is used whenever possible.

(c) Rework the predicate \texttt{fac/2} so that results are stored in the database (there should never be more than one entry in the database for each value) and are reused whenever possible. For example, suppose we make the following query:

\begin{verbatim}
?- fac(4,X).
X = 24.
true.
?- listing.
... facresult(4,24).
... true.
\end{verbatim}

Then, if we go on to ask \(?- fac(5,X).\) Prolog should not calculate everything anew, but should get the result for \texttt{fac(4,X)} from the database and multiply by 5.

¹http://www.swi-prolog.org/
Exercise 2 (6 Points, Powerset)

(a) Write a Prolog program $\text{subset}/2$ that is satisfied when the first argument is a subset of the second argument (that is, when every element of the first argument is a member of the second argument). For example:

```prolog
?- subset([a,b],[a,b,c])
true.
?- subset([b,d],[a,b,c])
false.
?- subset([], [a,b,c])
true.
```

Your program should be capable of generating all subsets of an input set by backtracking. For example, if you give it as input $\text{?- subset(X, [a,b,c])}$ it should successively generate all eight subsets of $[a,b,c]$.

(b) Using the subset predicate you have just written, and $\text{findall}/3$, write a predicate $\text{powerset}/2$ that takes a set as its first argument, and returns the powerset of this set as the second argument.

Exercise 3 (10 Points, Primes)

(a) A twin prime is a prime number $p$ such that one of $p + 2$ or $p - 2$ is also a prime. Write a predicate $\text{tprime}/1$ that is satisfied if the argument is a twin prime.

(b) Write a new predicate $\text{primef}/2$ that takes a nonnegative integer as its first argument and returns its prime factorization as second argument.

(c) Write a predicate $\text{tripleprime}/1$, as short as possible, that is satisfied if the argument $n$ is a prime, $n + 2$ is a prime and $n + 4$ is a prime.

(d) Goldbach’s conjecture says that every positive even number greater than 2 is the sum of two prime numbers. (Example: $34 = 3 + 31$.) It is a famous unsolved problem in number theory that is numerically confirmed for up to very large numbers. Write a predicate $\text{goldbach}/2$ to find (all) two prime numbers that sum up to a given even integer. Example:

```prolog
?- goldbach(34, L).
L = [3,31]; [5,29]; [11,23]; [17,17]
```

Exercise 4 (8 Points, Cut)

Consider the following Prolog program and predict the output to the queries below.

```prolog
top(X,Y):- predicate(X,Y).
top(X,X):- symmetric(X).
predicate(X,Y):- cutPoint(1), val4X(X), cutPoint(2), val4Y(Y).
predicate(X,Y):- symmetric(X), val4Y(Y).
val4X(a).
val4X(b).
val4Y(c).
val4Y(d).
symmetric(e).
cutPoint(_).
```

- `top(a,b)`
- `top(a,a)`
- `predicate(a,b)`
- `predicate(b,a)`
- `predicate(c,c)`
- `predicate(d,d)`
- `symmetric(e)`
- `cutPoint(1)`
- `cutPoint(2)`
(a) Enumerate all answers to the queries $\text{? - predicate}(X, Y)$ and $\text{? - top}(X, Y)$.
(b) What changes if cutPoint(1) is replaced with a cut?
(c) What changes if only cutPoint(2) is replaced with a cut?
(d) What are all answers to the queries $\text{? - predicate}(X, Y)$ and $\text{? - top}(X, Y)$ if both cutPoint(1) and cutPoint(2) are replaced with a cut?