TU Clausthal

Department of Informatics

Prof. Dr. J. Dix
Lic. F. Schlesinger

Artificial Intelligence

– Sheet 5: Neural Nets and Sentential Logic –

Date: 28. May 2014

Points:

_____ of 33

Group / Tutor:

Name(s) & Matr. no.:

Exercise 1 (10 Points, Perceptron)

Which of the following Boolean functions of three input variables (from $\{0,1\}^3$ into $\{0,1\}$, in particular $1+1=0, \bar{1}=0$ and $\bar{0}=1$) can be realized by a **single** perceptron? If it is possible to represent the function, determine the perceptron and calculate the corresponding weights. In the case the function is not representable, show why this is not the case.

1. $(x_1, x_2, x_3) \mapsto \bar{x}_1\bar{x}_2\bar{x}_3$,
2. $(x_1, x_2, x_3) \mapsto (x_1 + x_2)x_3$,
3. $(x_1, x_2, x_3) \mapsto \bar{x_1}\bar{x_3} + \bar{x_2}\bar{x_3}$,
4. $(x_1, x_2, x_3) \mapsto x_1x_2 + x_2x_3 + x_3x_1$.

Exercise 2 (3 Points, Tautologies)

State two tautologies that contain the propositions $Study$ and $Enjoy$. How many different tautologies can be built with these variables and the logical connectives? How many of these tautologies are equivalent to each other?

Exercise 3 (4 Points, Validity, truth, and satisfiability)

Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(a) $(New \rightarrow Good) \rightarrow New$

(b) $(New \rightarrow Good) \rightarrow (\neg Good \rightarrow \neg New)$

(c) $(Red \wedge \neg Round) \vee (Round \rightarrow Red)$

Exercise 4 (12 Points, Entailment)

In the following, you are given two knowledge bases involving unicorns. Consider for each of the knowledge bases the questions: (1) Is the unicorn mythical? (2) Is the unicorn magical? (3) Is the unicorn horned? Prove which of these properties hold and prove which of these properties do not hold (i.e. are not entailed).

(a) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal *or* a mammal, then it is horned. The unicorn is magical if it is horned.

(b) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is *either* immortal *or* a mammal, then it is horned. The unicorn is magical if it is horned.

To be submitted:

17. June 2014
in exercises' mailbox
(IfI building)

Exercise 5 (4 Points, Models)

Consider a vocabulary with *exactly* four variables, x_1, x_2, x_3 and x_4. How many models are there for the following sentences?

1. $\neg(\neg x_1 \vee x_1)$
2. $\neg(\neg x_1 \vee \neg x_2) \vee \neg(\neg x_2 \vee \neg x_3)$
3. $x_1 \leftrightarrow x_2 \leftrightarrow x_3$
4. $\neg(\neg((x_1 \wedge x_2) \rightarrow (x_1 \vee x_3 \vee x_4)) \vee \neg x_2)$