TU Clausthal

Department of Informatics

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Artificial Intelligence

– Sheet 6: Sentential Logic and Hoare Calculus –

Date: 17. June 2014

Points:

______________ of 30

Group / Tutor:

Name(s) & Matr. no.:

Exercise 1 (6 Points, Boolean connectives)

In the lecture, we have introduced the Boolean connectives $C := \{\neg, \vee, \wedge, \rightarrow\}$.

(a) Show that the Boolean connectives $\{\neg, \vee\}$ are sufficient. That is, show that each connective from C can be defined from just these two.

(b) Consider the binary Boolean function $\mid\, : \{0,1\}^2 \rightarrow \{0,1\}$ defined by $(1,1) \mapsto 0,\quad (0,1) \mapsto 1,\quad (1,0) \mapsto 1,\quad (0,0) \mapsto 0$. Can all Boolean connectives from C be defined by "|" ? Prove or disprove!

Exercise 2 (6 Points, Resolution)

(a) Given the theory $T = \{A \rightarrow \neg B,\ \neg A \rightarrow (C \wedge D),\ B\}$. Use resolution calculus to show that $T \models C$ holds. Explain your procedure!

(b) Apply the method described in (b) in order to decide whether the formula $(\neg A \rightarrow C) \wedge (C \rightarrow B) \wedge \neg A \wedge ((B \wedge C) \rightarrow A)$ is a tautology.

Exercise 3 (12 Points, Partial correctness)

Consider the following programs and use the annotation calculus to prove that the program is *partially* correct regarding the given pre and postconditions (note, that also the proof obligations have to be verified).

(a)
```
if (z > y) {
    x := y;
} else {
    x := z;
}
```
Precondition $\top$, postcondition $x = \min\{y, z\}$.

(b)
```
a := 0;
z := 1;
while (a < x) {
        z := z * y;
        a := a + 1;
}
```
Precondition $x \geq 0$, postcondition $z = y^x$.

Exercise 4 (6 Points, Total correctness)

Consider the following program Pow:

```
a := 0;
z := 1;
while (a < x) {
        z := z * y;
        a := a + 1;
}
```
Use the annotation calculus to prove that $\vdash^t \{x \geq 0\}$Pow$\{z = y^x\}$

To be submitted:

2. July 2014
until noon in the box