TU Clausthal

Department of Informatics

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Artificial Intelligence

– Sheet 7: FOL and Provers –

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Points:

________________ of 32

Group / Tutor:

Name(s) & Matr. no.:

To be submitted:

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before class

Exercise 1 (5 Points, Logical formalization)

Translate the following sentences into predicate logic. *Specify* the used predicate and function symbols.

1. Every rock band has a guitar player.
2. All new smarthphones have a GPS.
3. If Susan is on a diet, she does't eat any food that contains sugar.
4. The Germans beat some team, that the Spanish cannot beat.
5. Only fast cars are allowed on some highways.

Exercise 2 (6 Points, Interpretations)

Let the formula

$$\varphi \equiv \forall x \forall y \forall z (p(x, y) \wedge p(y, z) \rightarrow p(x, z))$$

be given.
Which of the following structures $A = (U, p^A)$ are models for φ? *Prove* or *disprove*!

1. $U = \mathbb{N}$, $p^A = \{(m, n) \mid m, n \in \mathbb{N}, m < n\}$,
2. $U = \mathbb{N}$, $p^A = \{(m, m + 1) \mid m \in \mathbb{N}\}$,
3. $U = 2^{\mathbb{N}}$, $p^A = \{(N, M) \mid N, M \subseteq \mathbb{N}, N \subseteq M\}$.

Exercise 3 (6 Points, Limitations of FOL)

Consider FOL with equality symbol "$\doteq$".

1. Show that for each natural number n, there is a formula Θ_n that is true in exactly the structures the universes of which consist of *at least* n elements.
2. There is no formula that is true *in exactly the structures with infinite universes*. Give a formal proof for this statement. (Hint: Compactness Theorem.)

Exercise 4 (3 Points, Expressiveness)

For a formula ϕ let $\exists!\varphi(x)$ mean that there exists a *unique* x such that $\varphi(x)$. Show that this quantifier is *definable*, i.e. rewrite the formula using only the usual operators and quantifiers from first-order logic with equality (i.e, $\exists$, $\forall$, $\doteq$, $\neg$, $\wedge$, and $\vee$).

Exercise 5 (4 Points, Resolution)

Use resolution to prove that the following clause set is unsatisfiable:

$$
\begin{aligned}
\{&\{P(g(b),b),\ Q(u,a),\ R(a,u,f(g(u)))\},\\
&\{Q(z,a),\ \neg P(g(b),z)\},\\
&\{R(a,y,f(x)),\ R(a,w,f(g(b))),\ \neg P(x,w)\},\\
&\{\neg R(a,z,f(g(b))),\ Q(y,a)\},\\
&\{\neg Q(v,a)\}\}
\end{aligned}
$$

Exercise 6 (4 Points, Resolution)

Sam, *Clyde* and *Oscar* are elephants. We know the following facts about them:

1. Sam is pink.
2. Clyde is gray and likes Oscar.
3. Oscar is either pink or gray (but not both) and likes Sam.

Use resolution to prove that a gray elephant likes a pink elephant; that is, prove $\exists x\, \exists y\, (Gray(x) \wedge Pink(y) \wedge Likes(x,y))$.

Exercise 7 (4 Points, Unification)

For each of the following sets, find the *most general unifier*, or justify that there is none.

1. $L_1 = \{P(x, g(x)), P(f(y), z), P(x, g(f(a)))\}$
2. $L_2 = \{Q(f(x), g(x)), Q(y, g(f(z))), Q(f(f(z)), u)\}$
3. $L_3 = \{R(x, f(y)), R(g(y, z), u), R(g(u, a), f(b))\}$
4. $L_4 = \{R(f(x, y, z)), R(f(g(a, y), h(x), a)\}$

Here u, x, y and z are variables, a and b constants, f, g and h function symbols and P, Q and R predicate symbols.